An Arithmetic Syllabus

for children aged 4 to 11

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This Syllabus is intended to provide both parents and teachers with clear step-by-step guidelines so that young children may learn the language of numbers without difficulty. In general, it should be left to the discretion of the teacher/parent to decide the extent to which concepts should be explained prior to teaching the method of calculation. Certainly, it is not necessary that children understand the meaning of 1 + 1 = 2, and other 'givens', in order to complete the sum in question. It is a general educational principle that children acquire knowledge before knowing why it is so. At a later time, the more precocious & more curious student may wish to discover how & why ideas & principles have acquired the objective status of knowledge.

THIS SYLLABUS SHOULD BE FOLLOWED WITHOUT THE USE OF CALCULATORS!

Calculators are best used <u>after</u> a child has mastered the basic arithmetic of this syllabus.

WHAT CHILDREN SHOULD BE TAUGHT
AT THE AGE OF FOUR ('RISING FIVES')

(These guidelines presuppose that it is appropriate to teach children at the age of four.)

- To identify the numbers 1 to 9; then the numbers 10, 20, 30 up to 90 in multiples of ten.
- To write down, from sight or from dictation, these numbers by forming the numerals correctly from top to bottom & with economical rotations. Children should hold pencils correctly.
- To recognise the addition sign (+); once this sign is familiar, to then recognise the subtraction sign (−).
- To recognise the equals sign (=). The earlier a child realises that the equals sign means 'the same as' & is not a symbol that denotes the answer, the better.
- To add two numbers together, as follows:

    $4 + 2 = 6$

    Then, add three numbers together, as follows:

    $4 + 1 + 1 = 6$

- To set out addition sums in columns & to add them correctly, as follows:

    $4 + 2 = 6 \qquad \rightarrow \qquad \begin{array}{r} 4 \\ +2 \\ \hline 6 \end{array}$

- After addition, to subtract one number from another, both along the line & in columns. Initially, all numbers should be less than ten.

- To complete sequences, as follows:

  1, 2, 3, ..., 5, 6

  2, 4, ..., 8, 10

- To recognise the different forms of language that denote addition (as in add, plus, more) & subtraction (as in subtract, minus, take away, less).

- To use a ruler to draw & measure straight lines.

- To arrange numbers in order from lower to higher & higher to lower.

- To identify the lowest or highest number from a random selection of numbers.

- To understand the use in simple sums of the term 'altogether' & then to answer orally, or by writing down the numeral, the answer to simple problems, as follows:

  Jack has six sweets. Anne has three sweets. How many sweets do they have altogether?

  Note: it is at this point that the child may first encounter the need to convert language into numbers.

- To identify simple shapes, such as squares, rectangles, triangles & circles.

To compare & sort objects, as follows:

- By length (that is, by width & height), from shorter to longer.
- By weight, from lighter to heavier.
- By touch, from softer to harder.
- By size, from smaller to larger (bigger).
- By colour, from lighter to darker.
- By sound, quieter to louder.
- By speed, from slower to faster.

Note: these forms of sorting objects according to a unit of measurement that they share in common should be accomplished by the appropriate use of 'the five senses', not by specific measurement.

From the start, children should be encouraged to enjoy mental arithmetic to develop their learning of this new language of numbers.

It is assumed that the teacher/parent will carry out regular assessments to confirm that the child has learned what he/she has been taught. Improvement & familiarity on the part of the young child should be reflected in his/her ability to perform the given tasks independently & with some speed.

WHAT CHILDREN SHOULD BE TAUGHT IN YEAR ONE AT THE AGE OF FIVE ('RISING SIXES')

(These guidelines presuppose that children are now proficient in the skills they have acquired in the preceding year. Teachers & parents who decide to begin the formal education of their children in Year One should begin at the beginning of this Arithmetic Syllabus on Page 1.)

- To identify the numbers 1 to 100, in & out of sequence.

- To write down, from sight or from dictation, these numbers by forming the numerals correctly from top to bottom & with economical rotations. Children should hold pencils correctly.

- To recognise that the same numeral has a different value depending on its position in the units, tens & hundreds column, as follows:

  > 33 → Although the numerals are the same, their values are different: the '3' to the right (in the units column) has the value of 3 units & the '3' to the left (in the tens column) has the value of 30 (3 tens). This introduces the child to the necessity of writing numbers in columns.

- To add & subtract 'easy' numbers in lines & columns.

- To set out in columns addition & subtraction sums that are given across the line. Note: it is not necessary to write the numbers in order of size; however, it is essential that digits be placed <u>clearly</u> in their correct columns, as follows:

  $$41 + 57 = 98 \qquad \rightarrow \qquad \begin{array}{r} 41 \\ +57 \\ \hline 98 \end{array}$$

- To practise the different ways by which numbers may be added or subtracted to give the same answer, as follows:

    7 + 21 = 28   39 − 11 = 28

    22 + 6 = 28   29 − 1 = 28

    28 = 1 + 2 + 25   28 = 99 − 71

- To recognise odd & even numbers.
- To identify & sketch simple 2-dimensional shapes.
- To carry out simple measurements with a ruler.
- To introduce other units of linear measurement such as: hands, feet, pieces of string or any repeatable unit. Children may now learn that objects may be measured by any fixed unit of choice. Hence, the teacher or parent may wish to discuss the more appropriate units of measurement & why these have become standardised.
- To complete sequences, particularly with higher numbers, as follows:

    11, 13, …., 17, 19

    20, 30, …., 50, 60, …., 80, 90

    3, 5, 8, 12, …., 23, 30, 38

- To tell the time, by hours & half-hours, on an analogue time-piece.
- To find the answer to numerical problems that requires addition or subtraction of numbers from 1 to 100, as follows:

    i) Fred likes marbles. He already has 45 marbles. His friend, James, gives him 12 marbles as a birthday present. How many marbles does Fred have altogether?

    ii) Mandy has too many marbles. She has 99 marbles

altogether. So, she gives 25 marbles to her friend, Ruby. How many marbles does Mandy have left?

- To recognise & identify coins in current use. Then, to answer simple money problems, as follows:

   Jenny has 20p in her money box. Her Mummy gives her a ten-penny piece & a two-penny piece. How much money does Jenny have now?

- To recognise the multiplication sign (×); see the following note.

Note: it is now appropriate for children to learn the easiest of arithmetical tables, namely, the 2× table, the 10× table & the 5× table. It is absolutely imperative that arithmetical tables are learned as a complete statement, not simply by reciting the answer in sequence. Therefore, the child must say 4 × 2 = 8, 5 × 2 = 10, 6 × 2 = 12, & so on. He/she must <u>not</u> count out 2, 4, 6, 8, 10, & so on. The practice of learning tables at speed is of such value that the teacher/parent should give this exercise due importance. In the same way, adding & subtracting at speed, without writing numbers down, is of the greatest value. Needless to say, at this stage mental arithmetic should be confined to the numbers 1 to 100, but 'without borrowing'. Ideally, tables should go up to the multiplication of numbers up to 12, so that children learn 11 × 2 = 22 & 12 × 2 = 24.

- To set out along the line multiplication sums using the above tables, as follows:

    $4 \times 2 = 8$

    $10 \times 5 = 50$

As before, mental arithmetic should play an important role in the acquisition of number skills.

It is assumed that the teacher/parent will carry out regular assessments to confirm that the child has learned what he/she has been taught. Improvement & familiarity on the part of the young child should be reflected in his/her ability to perform the given tasks independently & with some speed.

WHAT CHILDREN SHOULD BE TAUGHT IN YEAR TWO AT THE AGE OF SIX ('RISING SEVENS')

(These guidelines presuppose that children are now proficient in the skills they have acquired in the preceding years.)

- To identify the numbers 1 to 1000, in & out of sequence.

- To write down, from sight or from dictation, these numbers by forming the numerals correctly from top to bottom & with economical rotations. Children should hold pencils correctly.

- To identify numerical value according to the position (that is, the column) of the number, as follows:

    333 → Although the numerals are the same, their values are different: the '3' in the units column has the value of 3 units, the '3' in the tens column has the value of 30 (3 × ten) & the 3 in the hundreds columns has the value of 300 (3 × a hundred).

- To add in columns ('without carrying' & 'with carrying') & subtract in columns ('without borrowing' & 'with borrowing'), as follows:

    431   697
    +527   – 243
     958   454

    486   612
    +135   – 248
     621   364

Note: when adding 'with carrying', the convention is to indicate the amount to be 'carried over' underneath the sum, in the correct column; when subtracting 'with borrowing', the convention is to borrow 'from the top' from the adjoining column on the left. It is extremely helpful, at this stage, for the child to write on squared paper so that columns may be clearly set out.

- To continue to practise the different ways by which larger numbers may be added or subtracted to give the same answer.

- To recognise the division sign (÷) & set out division sums, without remainders, only using arithmetical tables learned so far, as follows:

  $$6 ÷ 2 = 3$$
  $$25 ÷ 5 = 5$$

- Now that the child can tell the time by hours & half-hours, to proceed to tell the time by quarter-hours, when the time may be indicated before or after the hour, as follows:

  i) a quarter past three

  ii) a quarter to three

  Likewise, the child may now learn to tell the time according to how many minutes before or after the hour, as follows:

  i) five minutes past three

  ii) five minutes to three

- To recognise & extract information from pictograms & simple bar charts.

- To complete sequences, particularly with higher numbers, now from 1 to 1000.

- To learn, by heart, that there are:

  i) 100 pennies in a pound

  ii) 100 centimetres in a metre

  iii) 1000 grams in a kilogram

  iv) 1000 millilitres in a litre

  Note: children should now learn the context in which different units are used.

- To measure length in centimetres & metres (with a ruler); to measure capacity in millilitres & litres (with a measuring jug); & to measure weight in grams & kilograms (on scales). Then, to estimate the same.

- To find the answer to simple money problems, simple time problems & simple problems of weights & measures.

- Now that children can identify simple 2-dimensional shapes (in Year One), to sketch & draw them accurately.

- To improve all forms of mental arithmetic, including arithmetical tables, in accordance with the syllabus so far.

Note: after learning, in Year One, the 2× table, the 10× table & the 5× table, it is now appropriate to learn the 11× table, the 3× table & the 4× table. Once again, it is absolutely imperative that arithmetical tables are learned as a complete statement, not simply by reciting the answer in sequence. The practice of learning tables at speed is of such value that the teacher/parent should give this exercise due importance.

In the same way, adding & subtracting at speed, without writing numbers down, is of the greatest value. Needless to say, at this stage mental arithmetic should be confined to the numbers 1 to 1000, still 'without borrowing'. Ideally, tables should go up to the multiplication of numbers up to 12, so that children learn 11 × 3 = 33 & 11 × 11 = 121, & so on in relation to other tables too.

It is assumed that the teacher/parent will carry out regular assessments to confirm that the child has learned what he/she has been taught. Improvement & familiarity on the part of the young child should be reflected in his/her ability to perform the given tasks independently & with some speed.

WHAT CHILDREN SHOULD BE TAUGHT IN YEAR THREE AT THE AGE OF SEVEN ('RISING EIGHTS')

(These guidelines presuppose that children are now proficient in the skills they have acquired in the preceding years.)

- To identify & write the numbers, from sight & from dictation, 1 to 1,000,000 in & out of sequence.
- To identify numerical value according to the position (that is, the column) of the number, as follows:

  333,333 → Although the numerals are the same, their values are different: the '3' in the units column has the value of 3 units, the '3' in the tens column has the value of 30 (3 × ten), the 3 in the hundreds columns has the value of 300 (3 × a hundred), the 3 in the thousands column has the value of 3000 (3 × a thousand), the 3 in the ten thousands column has the value of 30,000 (3 × ten thousand) & the 3 in the hundred thousands column has the value of 300,000 (3 × a hundred thousand).

- To add in columns ('without carrying' & 'with carrying') & subtract in columns ('without borrowing' & 'with borrowing'), as follows:

$$
\begin{array}{r}
4131 \\
+5527 \\
\hline
9658
\end{array}
\qquad
\begin{array}{r}
6597 \\
-2343 \\
\hline
4254
\end{array}
$$

```
  47986              67512
+ 1635            −  2748
  49621              67764
```

- To multiply numbers, along the line, with arithmetical tables already learned, as follows:

  7 × 4 = 28
  10 × 11 = 110

- Now, to introduce 'short multiplication' in columns (with & without the carrying over of numbers to the next column) by making use of arithmetical tables that have been learnt so far, as follows:

```
  234                234
×   2              ×   5
  468               1170
```

- To introduce 'short division' in columns, as follows:

  simple short division →   444
                                 2)888

  short division with internal remainder →   449
                                              2)898

  short division with external remainder →   444 r 1
                                              2)889

- To know, by the end of Year Three, all arithmetical tables up to **12×** table, in & out of sequence & at speed.

Note: arithmetical tables should be reversible by division, as follows:

   $6 \times 9 = 54$

   $54 \div 9 = 6$

   $54 \div 6 = 9$

- To recognise the different uses of language that denote division, as follows:

   $6 \div 2$ is the same as i) six divided by two, or ii) how many twos in six? or iii) two into six. **It is imperative that children not confuse these!**

- To tell the time, from an analogue time-piece, making use of am & pm, & then, to solve simple problems that make use of this notation.

It is now appropriate to find the answers to questions such as these:

o If one pen costs 64 pence, how much will eleven pens cost?

o If seven stamps cost 84 pence, how much will one stamp cost?

o What is the total cost of:

   5 pencils at £1.00 each

   7 rubbers at £1.50 each

   11 rulers at 75 pence each

   20 stickers at 9 pence each

- To balance equations, as follows:

$$6 \times \square = 3 \times 8$$

$$24 \div 2 = 3 \times \square$$

- To make the smallest & largest number, both even & odd, from a random selection of numbers, as follows:

  Make the largest possible even number with the following numbers: 8 5 9 3. The answer is 9,538.

- To convert measurements from one unit to another, as follows:

  metres $\rightarrow$ kilometres

  metres $\rightarrow$ centimetres

  millilitres $\rightarrow$ litres

  grams $\rightarrow$ kilograms

- To solve simple problems using weights & measures.

- To introduce fractions, by pie charts, divisions of length & time, & by everyday examples.

- To learn the points of the compass.

- To learn how to use a compass & protractor.

- To improve all forms of mental arithmetic, including arithmetical tables, in accordance with the syllabus so far.

It is assumed that, as before, the teacher/parent will carry out regular assessments to confirm that the child has learned what he/she has been taught. Improvement & familiarity on the part of the young child should be reflected in his/her ability to perform the given tasks independently & with some speed.

WHAT CHILDREN SHOULD BE TAUGHT IN YEAR FOUR AT THE AGE OF EIGHT ('RISING NINES')

(These guidelines presuppose that children are now proficient in the skills they have acquired in the preceding years.)

- To consolidate & make fluent addition, subtraction, short multiplication & short division.
- To perform long multiplication, whereby the multiplier is higher than 12. Initially, long multiplication should be confined to two-digit multipliers, as follows:

$$
\begin{array}{r}
751 \\
\times\ 23 \\
\hline
2253 \\
15020 \\
\hline
17273 \\
\hline
\end{array}
$$

Notes:

It is best to first multiply by the number in the units column & then by the number in the tens column, before adding them together. Before thinking of the multiplier as a compound that comprises two or more multipliers whereby all long multiplication is reduced to sequences of short multiplication, children should learn to multiply by the given multiplier as a single operation, as in the example above.

- To perform long division, whereby the divisor is higher than 12. Initially, long division should be confined to quotients with or without remainders & to two-digit divisors, as follows:

```
       19 r 12
   23)449
       23
       219
       207
         12
```

Note: it is imperative that children show their working-out in the form of short multiplication sums until they find the number nearest to the dividend; therefore, in the above example, there should be working-out, in the form of multiplication sums to confirm or reject estimates, to find the quotient that is nearest to **44** & is nearest to **219**.

- To introduce perimeter & area, by calculation, of squares & rectangles.
- To convert times in the 12-hour clock to times in the 24-hour clock.
- To add & subtract time.
- To find the average of a set of numbers.
- The use of a decimal point to further develop the idea of place value $\rightarrow$ to recognise that the same numeral has a different value depending on its position in relation to the decimal point,

as follows:

> 345.67 → The '5' is in the units column & has the value of 5 units; the '4' is in the tens column & has the value of 40 (4 × ten); the '3' is in the hundreds column & has the value of 300 (3 × a hundred); the '6' is in the one-tenth column & has the value of $^6/_{10}$ (6 × $^1/_{10}$); the '7' is in the one-hundredth column & has the value of $^7/_{100}$ (7 × $^1/_{100}$). Once again, the importance of columns in arithmetic must be reinforced.

- To solve problems of money, distance, length, capacity & time.
- To improve all forms of mental arithmetic, including arithmetical tables, in accordance with the syllabus so far.

It is assumed that, as before, the teacher/parent will carry out regular assessments to confirm that the child has learned what he/she has been taught. Improvement & familiarity on the part of the young child should be reflected in his/her ability to perform the given tasks independently & with some speed.

WHAT CHILDREN SHOULD BE TAUGHT IN YEAR FIVE AT THE AGE OF NINE ('RISING TENS')

(These guidelines presuppose that children are now proficient in the skills they have acquired in the preceding years.)

- To consolidate & make fluent long multiplication & long division.
- To find areas of compound shapes made up of squares & rectangles, & to find areas of triangles ($\frac{1}{2}$ base × perpendicular height).
- To find volumes of cubes & cuboids.
- Equivalent fractions, as follows:

$$\frac{1}{4} = \frac{2}{8} = \frac{4}{16} = \frac{20}{80} = \frac{200}{800}$$

- To simplify fractions, as follows:

$$\frac{200}{800} = \frac{20}{80} = \frac{4}{16} = \frac{2}{8} = \frac{1}{4}$$

- To add, subtract, multiply & divide fractions, as follows:

  * **addition** → $\frac{1}{5} + \frac{3}{5} = \frac{4}{5}$ (denominators are the same)

    → $\frac{1}{5} + \frac{3}{4} = \frac{4}{20} + \frac{15}{20}$

    $= \frac{19}{20}$ (different denominators)

    → $2\frac{1}{5} + 3\frac{3}{4} = 2\frac{4}{20} + 3\frac{15}{20}$

    $= 5\frac{19}{20}$

  * **subtraction** → $\frac{3}{5} - \frac{1}{5} = \frac{2}{5}$

    → $\frac{3}{4} - \frac{1}{5} = \frac{15}{20} - \frac{4}{20}$

    $= \frac{11}{20}$

    → $3\frac{1}{5} - 1\frac{3}{4} = 3\frac{4}{20} - 1\frac{15}{20}$

    $= 2\frac{24}{20} - 1\frac{15}{20}$

    $= 1\frac{9}{20}$

  * **multiplication** → $\frac{1}{5} \times \frac{3}{4} = \frac{3}{20}$

$$\rightarrow \quad 6\tfrac{2}{3} \times 1\tfrac{8}{25} = \tfrac{20}{3} \times \tfrac{33}{25}$$

$$= \tfrac{4}{1} \times \tfrac{11}{5} \text{ (by simplification)}$$

$$= \tfrac{44}{5}$$

$$= 8\tfrac{4}{5}$$

* **division** $\rightarrow \tfrac{1}{5} \div \tfrac{3}{5} = \tfrac{1}{5} \times \tfrac{5}{3}$

$$= \tfrac{1}{1} \times \tfrac{1}{3} \text{ (by simplification)}$$

$$= \tfrac{1}{3}$$

$$\rightarrow 3\tfrac{8}{9} \div 2\tfrac{6}{7} = \tfrac{35}{9} \div \tfrac{20}{7}$$

$$= \tfrac{35}{9} \times \tfrac{7}{20}$$

$$= \tfrac{7}{9} \times \tfrac{7}{4} \text{ (by simplification)}$$

$$= \tfrac{49}{36}$$

$$= 1\tfrac{13}{36}$$

- To add, subtract, multiply & divide decimals, as follows:

    * **addition** $\rightarrow 45.76 + 25.944 = 71.704$ (see working-out below)

    $$45.76$$
    $$+\ \underline{25.944}$$
    $$\underline{71.704}$$

    * **subtraction** $\rightarrow 45.7 - 26.811 = 18.899$ (see working-out below)

    $$45.7$$
    $$-\ \underline{26.811}$$
    $$\underline{18.899}$$

    * **multiplication** $\rightarrow 0.1 \times 0.3 = 0.03$

    $$\rightarrow 1.23 \times 1.3 = 1.599$$ (see working-out below)

    $$123$$
    $$\times\ \underline{13}$$
    $$\underline{1599}$$

    * **division** $\rightarrow 1.38 \div 0.3 = \tfrac{1.38}{0.3}$

$$= \frac{13.8}{3}$$

$$= \quad \underline{4.6}$$

$$3)13.8$$

- To convert fractions to decimals & decimals to fractions, as follows:

    $\frac{11}{1000} = 0.011$

    $0.01 = \frac{1}{100}$

- To convert fractions to percentages, as follows:

    $\frac{1}{5} = \frac{20}{100}$

    $\quad = 20\%$

- To use percentages, as follows:

    Find 27% of 220

    Method (1):   100% = 220

    10% = 22

    1% = 2.2

    5% = 11

    20% = 44

    2% = 4.4

    So, 27% = 20% + 5% + 2%

    = 44 + 11 + 4.4

    = 59.4

    Method (2): $\frac{27}{100} \times 220 = \frac{27}{100} \times \frac{220}{1}$

    $= \frac{27}{10} \times \frac{22}{1}$

    $= \frac{27}{5} \times \frac{11}{1}$

$$= {}^{297}/_5$$
$$= 59^2/_5$$

- To solve problems expressed in terms of fractions, decimals or percentages.
- To solve more advanced problems of money, distance, length, capacity & time.
- To extract information from charts & graphs.
- To round numbers to the nearest ten, hundred, thousand, etc.
- To improve all forms of mental arithmetic, including arithmetical tables, in accordance with the syllabus so far.

It is assumed that, as before, the teacher/parent will carry out regular assessments to confirm that the child has learned what he/she has been taught. Improvement & familiarity on the part of the young child should be reflected in his/her ability to perform the given tasks independently & with some speed.

Furthermore, children should now be introduced to formal objective testing within a time limit.

WHAT CHILDREN SHOULD BE TAUGHT IN YEAR SIX AT THE AGE OF TEN ('RISING ELEVENS')

(These guidelines presuppose that children are now proficient in the skills they have acquired in the preceding years.)

- To revise fractions, decimals & percentages.

- To find lines of symmetry.

- To sketch nets of 3-dimensional shapes.

- To introduce very simple probability, as follows:

  * What is the probability of spending your summer holidays on the planet Saturn?

  * What is the probability of picking out 'a queen' from a pack of cards?

- To encounter more challenging forms of mental arithmetic that make use of what has been learned so far.

Note: In the main, the syllabus for Year Six is a consolidation of everything that has been taught to date. In particular, this should be a time for the pupil to apply what was learned in Year Five to more difficult & more challenging problems. That is to say, it is in Year Six that the more advanced pupil proceeds from arithmetic to mathematics!

The degree to which the pupil may be said to have mastered the primary school curriculum should be reflected in his/her ability to perform the given tasks independently & with some speed.

The practice of formal objective testing within a time limit should continue.